NETWORK-ANALYZER SELF-CALIBRATION WITH FOUR OR FIVE STANDARDS FOR THE 15-TERM ERROR-MODEL

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Abstract

Novel self-calibration equations for the 15-term error-model are presented that form an analogon to the well known similarity-transforms of the 7-term error-model ([3], [4]). These equations permit construction of a multitude of self-calibration procedures, one of which, the Tmrg-procedure, is introduced.

Introduction

Calibration of four-receiver vector network-analyzers (VNA), according to one of the established error-models ([1], [2], [4]) requires precisely known standards. Application of self-calibration procedures relaxes the requirements, imposed upon the standards, and allows for partially unknown standards. General formulas for the construction of self-calibration procedures exist, so far, only for the 7-term error-model [4]. As far as the 15-term model [1], [5] that also accounts for leakage-errors is concerned, all self-calibration procedures known today rely on one standard being represented by the zero-matrix [6] (i.e. using S- parameters, the ideal double match) or use iterative methods [7].

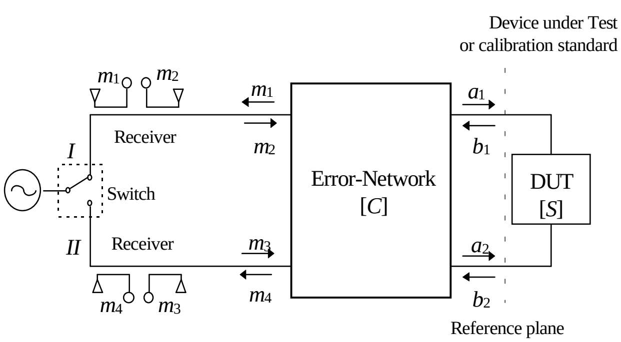

Figure 1: A four-receiver VNA with coupled reflectometers (15-term error-model)

A general, closed-form relation between measured values of the calibration standards and the standards' S-parameters is not known to date and will be presented in this paper.

The 15-Term Error-Model

Following the derivation of the error-model in [1], or [5], the four-port [C] in figure 1 relates the waves at the DUT to the measured values m_i (i=1..4) and can be partitioned into four 2x2 matrices,

$$\begin{pmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{pmatrix} = [C] \begin{pmatrix} m_1 \\ m_4 \\ m_2 \\ m_3 \end{pmatrix} = \begin{pmatrix} [G] & [E] \\ [F] & [H] \end{pmatrix} \begin{pmatrix} m_1 \\ m_4 \\ m_2 \\ m_3 \end{pmatrix}$$

yielding a compact representation of the error-model

$$[G][M_n]+[E]=[S_n]\left([F][M_n]+[H]\right) \qquad (1)$$

with the measurement-matrix [M_n] and the scattering-matrix [S_n] of the standard given as

$$[M_n] = \begin{pmatrix} m_1' & m_1'' \\ m_4' & m_4'' \end{pmatrix} \begin{pmatrix} m_2' & m_2'' \\ m_3' & m_3'' \end{pmatrix}^{-1}, \quad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = [S_n] \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}.$$

The single and double strokes denote the measured values in the respective position of the switch.

Calibration of the 15-term error-model requires five calibration measurements (n = 1..5), resulting in an overdertermined system of 20 equations in the 15 unknown error-terms. Self-calibration uses this redundancy to determine up to *five* unknown parameters of the calibration-standards *before* the error-terms are computed. The needed relation between the measured values and the standards' S-parameters,

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eliminating the (yet unknown) error-terms [E] .. [H], will now be derived.

Subtracting the j-th calibration measurement from the i-th yields

$$\begin{aligned} G(M_i - M_j) &= S_i F M_i + S_i H - S_j F M_j - S_j H \\ &\quad + (S_j F M_i - S_j F M_i) \\ &= S_j F(M_i - M_j) + (S_i - S_j)(H + F M_i) \end{aligned} \quad , \qquad (2)$$

where adding $0 = S_j F M_i - S_j F M_i$ provides an advantageous way of combining terms. In a similar fashion, measurement j and k ($i \neq j \neq k$) are combined:

$$G - S_j F = (S_j - S_k)(H + F M_k)(M_j - M_k)^{-1} \qquad (3)$$

Equating (2) and (3) results in:

$$\begin{aligned} &(S_i - S_j)(H + F M_i)(M_i - M_j)^{-1} \\ &\quad = (S_j - S_k)(H + F M_k)(M_j - M_k)^{-1} \end{aligned} \qquad (4)$$

Introducing the notation

$$\Delta S_{m,n} = S_m - S_n \;\; , \;\; \Delta M_{m,n} = M_m - M_n \;\; , \;\; m, n = 1..5$$

and solving equation (4) for $(H + F M_i)$ yields

$$(H + F M_i) = \Delta S_{i,j}^{-1} \Delta S_{j,k} (H + F M_k) \Delta M_{j,k}^{-1} \Delta M_{i,j} \qquad (5)$$

The fourth calibration measurement (denoted with l) is now used to create a similar equation, such that the term $(H + F M_i)$ remains unchanged

$$(H + F M_i) = \Delta S_{i,l}^{-1} \Delta S_{l,k} (H + F M_k) \Delta M_{l,k}^{-1} \Delta M_{i,l} \qquad (6)$$

Equating (5) and (6) yields the similarity transform

$$\begin{aligned} &\Delta M_{j,k}^{-1} \Delta M_{i,j} \Delta M_{i,l}^{-1} \Delta M_{l,k} \\ &\quad = (H + F M_k)^{-1} \Delta S_{j,k}^{-1} \Delta S_{i,j} \Delta S_{i,l}^{-1} \Delta S_{l,k} (H + F M_k) \end{aligned} \; , \quad (7)$$

which results in the two nonlinear relations

$$\begin{aligned} &Trace\left\{ \Delta M_{j,k}^{-1} \Delta M_{i,j} \Delta M_{i,l}^{-1} \Delta M_{l,k} \right\} \\ &\qquad = b_1 = Trace\left\{ \Delta S_{j,k}^{-1} \Delta S_{i,j} \Delta S_{i,l}^{-1} \Delta S_{l,k} \right\} \\ &Det\left\{ \Delta M_{j,k}^{-1} \Delta M_{i,j} \Delta M_{i,l}^{-1} \Delta M_{l,k} \right\} \\ &\qquad = b_2 = Det\left\{ \Delta S_{j,k}^{-1} \Delta S_{i,j} \Delta S_{i,l}^{-1} \Delta S_{l,k} \right\} \end{aligned} \qquad (8)$$

between the measured values [M_n] and the standards' S-parameters (Invariance of the eigenvalues of similar matrices).

As only four calibration measurements are used by equation (7), the fifth standard [S_m], [M_m] may be substituted into (7) to yield two more similarity transformations:

$$\begin{aligned} &\Delta M_{j,k}^{-1} \Delta M_{m,j} \Delta M_{m,l}^{-1} \Delta M_{l,k} \\ &\quad = (H + F M_k)^{-1} \Delta S_{j,k}^{-1} \Delta S_{m,j} \Delta S_{m,l}^{-1} \Delta S_{l,k} (H + F M_k) \end{aligned} \quad (9)$$

(i-th standard replaced by m-th standard),

$$\begin{aligned} &\Delta M_{m,k}^{-1} \Delta M_{i,m} \Delta M_{i,l}^{-1} \Delta M_{l,k} \\ &\quad = (H + F M_k)^{-1} \Delta S_{m,k}^{-1} \Delta S_{i,m} \Delta S_{i,l}^{-1} \Delta S_{l,k} (H + F M_k) \end{aligned} \quad (10)$$

(j-th standard replaced by m-th standard).

Together with (8), the resulting trace- and determinant-equalities provide six nonlinear self-calibration equations, sufficient for computation of the maximally five unknowns that the 15-term error-model allows for.

It is interesting to note that the derivation of (7) holds as long as the structure of the error model (1) is unchanged. DUT and/or the measurement matrix [M] may therefore also be expressed in T- (transmission) parameters or even in chain-parameters, using voltages and currents instead of waves. Even the extension to the error-model of a N-port VNA, as described in [1] is straight forward. In this case all quadrants of the error-matrix, the standards' S- (T-) matrix and the measurement-matrices are NxN-matrices. The invariance of the eigenvalues of the two similar matrices in (7) yields N relations between the measured values and the standards.

The Tmrg-procedure

The general nature of (8) allows the construction of self-calibration procedures to optimally match the needs of a specific application. The following example addresses coaxial calibration of a 2-port VNA and is the first closed-form 15-term calibration-procedure with a completely consistent set of calibration standards.

Constructing the five standards from a Through and three reflection one-ports with reflection-coefficients m, r and g, the first four standards suffice for the determination of the unknown reflection-coefficients r and g:

$$S_T^{(k)} = \begin{pmatrix} 0 & t \\ t & 0 \end{pmatrix} \quad , \quad S_{rm}^{(j)} = \begin{pmatrix} r & 0 \\ 0 & m \end{pmatrix} \quad ,$$

$$S_{mg}^{(l)} = \begin{pmatrix} m & 0 \\ 0 & g \end{pmatrix} \quad , \quad S_{gr}^{(i)} = \begin{pmatrix} g & 0 \\ 0 & r \end{pmatrix} \quad , \quad S_{rg}^{(m)} = \begin{pmatrix} r & 0 \\ 0 & g \end{pmatrix}$$

Substituting the standards i, j, k, and l into equation (8), and evaluating the product of S-matrix differences yields

$$\Delta S_{j,k}^{-1} \Delta S_{i,j} \Delta S_{i,l}^{-1} \Delta S_{l,k} =$$

$$B = \frac{1}{t^2 - r\,m} \begin{pmatrix} t^2 C_1 - m^2 C_2 & t\left(mC_2 - gC_1\right) \\ t\left(rC_1 - mC_2\right) & t^2 C_2 - r\,g\,C_1 \end{pmatrix}$$

with

$$C_1 = \frac{r-m}{r-g} \quad , \quad C_2 = \frac{g-r}{g-m} \, ,$$

resulting in

$$Trace\{\Delta M_{j,k}^{-1} \Delta M_{i,j} \Delta M_{i,l}^{-1} \Delta M_{l,k}\} = Trace(B)$$

$$= b_1 = \frac{m^2(g-m) + g\,r\,(m-r) + t^2(r-g)}{\left(r\,m - t^2\right)(m-g)} \qquad (11)$$

$$Det\{\Delta M_{j,k}^{-1} \Delta M_{i,j} \Delta M_{i,l}^{-1} \Delta M_{l,k}\} = Det\,(B)$$

$$= b_2 = \frac{\left(t^2 - m\,g\right)(r-m)}{\left(t^2 - r\,m\right)(m-g)} \qquad (12)$$

Equation (12) can be solved to yield a linear relation for r

$$r = \frac{b_2 t^2 (g-m) + m\left(g\,m - t^2\right)}{b_2 m (g-m) + \left(g\,m - t^2\right)} \quad , \qquad (13)$$

and can be combined with (11) to form a quadratic equation for g.

$$g^2 - g\,\frac{2m\,t^2 a_1 a_2}{m^2 a_3 - t^2 b_2^2} + \frac{t^4 a_3 - t^2 m^2 b_2^2}{m^2 a_3 - t^2 b_2^2} = 0 \qquad (14)$$

with

$$a_1 = b_1 - 1 - 2b_2 \quad , \quad a_2 = b_2 + 1$$

$$a_3 = (b_1 - 1)(b_2 + 1) - b_2(2 + b_2)$$

Choosing the proper root for g requires knowledge about the sign of that reflection standard. Using a short for g and an open for r provides the necessary sign information and makes the standards sufficiently distinct for subsequent use as fully known calibration standards.

The only parameters that must be known are the transmission coefficient t of the T-standard and the reflection m. The quantity m should be small for numerical reasons and must be known, but the standard is not required to be an *ideal* match.

Since only *one* set of reflection one-ports is physically required, the postulated equality of the reflection-coefficients that enter the different standards is guaranteed. This is regarded as a major improvement over self-calibration techniques, that require the same reflection coefficient to be connected to both ports simultaneously.

Measurement Results

The Tmrg-procedure was tested in a coaxial environment with a HP8510C network analyzer as depicted in figure 2. Artificial leakage is introduced by the 20dB attenuator, connecting the analyzer's ports via T-splitters.

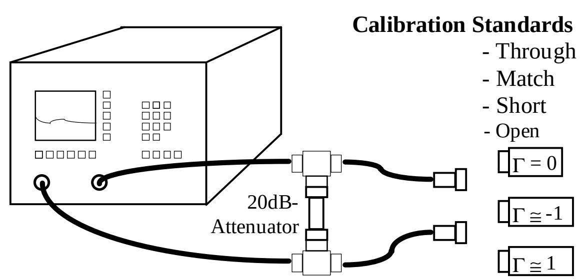

Figure 2. Measurement setup of the Network Analyzer with artificial Crosstalk

Even with this high amount of cross-talk, the Tmrg-procedure performs well, recovering the data of a measured 25Ω air-line and a 20dB attenuator as depicted in figure 4 through 6. Figure 3 displays the basis for the 15-term calibration, namely the self-calibration result of the open's and the short's phase.

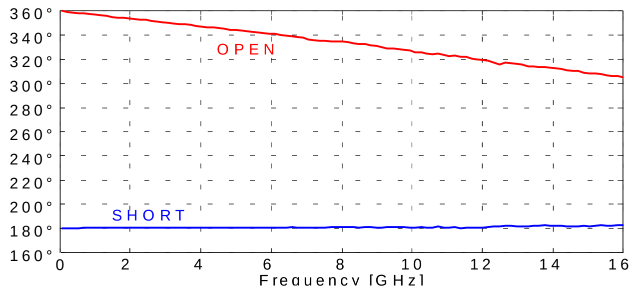

Figure 3. Phase of Open and Short as computed by self-calibration (eqn. 13 and 14)

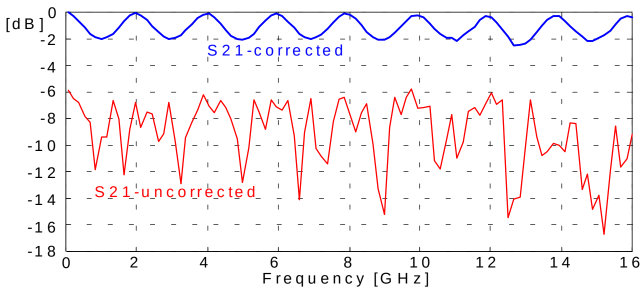

Figure 4. Transmission of the measured 25Ω air-line (corrected / uncorrected)

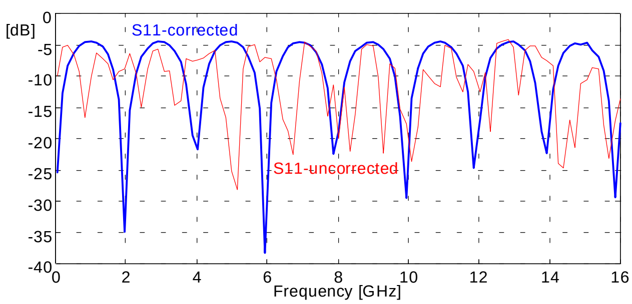

Figure 5. Reflection of the measured 25Ω air-line (corrected / uncorrected)

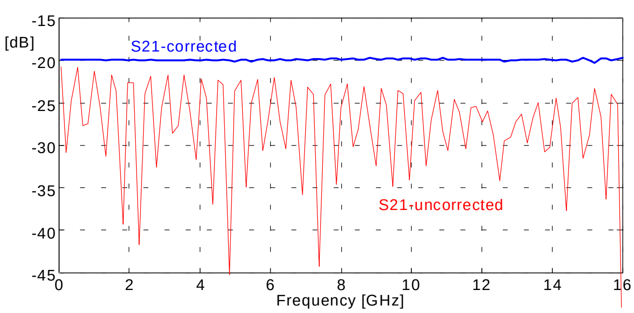

Figure 6. Transmission of the measured 20dB-attenuator (corrected / uncorrected)

Conclusion

The presented self-calibration equations for the 15-term error-model fill a gap that existed in comparison to the 7-term error-model. The data of the five calibration measurements can be used to determine up to five unknown parameters of the calibration standards by exploiting six nonlinear equations stemming from similarity-transformations. As an example, geared towards coaxial measurements, the Tmrg-procedure is presented that, besides the Through-standard (of *known* transmission factor) requires only one *known* (and two unknown) reflection one-ports as standards. Inconsistency errors are avoided by constructing the four reflection-standards from those three reflection one-ports.

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